Quantitative Aptitude

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Part A: Verbal Reasoning

Section-D: Numerical Ability

CHAPTER 1

NUMBER SYSTEM

NUMBER SYSTEM

A number system relates quantities and symbols. The base or radix of a number system represents the number of digits or basic symbols in that particular number system.

Decimal is a base (or radix) 10 numeral system. This means that the system has ten symbols or numerals to represent any quantity. These symbols are called Digits. The ten symbols are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0.

Types of Numbers :

Real numbers: Real numbers comprise the full spectrum of numbers. They can take on any form – fractions or whole numbers, decimal points or no decimal points. The full range of real numbers includes decimals that can go on forever and ever without end.

For Example: 8, 6, $2+\sqrt{3}$, $\frac{3}{5}$ etc.

Natural numbers: A natural number is a number that comes naturally. Natural Numbers are counting numbers from 1, 2, 3, 4, 5, ...

Whole numbers: Whole numbers are just all the natural numbers plus zero.

For Example: 0, 1, 2, 3, 4, 5, and so on upto infinity.

Integers: Integers incorporate all the qualities of whole numbers and their opposites (or additive inverses of the whole numbers). Integers can be described as being positive and negative whole numbers.

For Example: … –3, –2, –1, 0, 1, 2, 3, . . .

Rational numbers: All numbers of the form $\frac{p}{q}$ where p and q are integers ($q \neq 0$) are called Rational numbers.

For Example: $4, \frac{3}{4}, 0, ...$

Irrational numbers: Irrational numbers are the opposite of rational numbers. An irrational number cannot be written as a fraction, and decimal values for irrational numbers never end and do not have a repeating pattern in them. *'pi'* with its never-ending decimal places, is irrational.

For Example: $\sqrt{7}, \sqrt{5}, 2+\sqrt{2}, \pi, ...$

Even numbers: An even number is one that can be divided evenly by two leaving no remainder, such as 2, 4, 6, and 8.

Odd numbers: An odd number is one that does not divide evenly by two, such as 1, 3, 5, and 7.

Prime numbers: A prime number is a number which can be divided only by 1 and itself. The prime number has only two factors, 1 and itself.

For example: 2, 3, 5, 7, 11, 13, 17, ... are prime numbers.

Composite Number: A Composite Number is a number which can be divided into a number of factors other than 1 and itself . Any composite number has additional factors than 1 and itself.

For example: 4, 6, 8, 9, 10 .....

Co-primes or Relatively prime numbers: A pair of numbers not having any common factors other than 1 or –1. (Or alternatively their greatest common factor is 1 or –1)

For Example: 15 and 28 are co-prime, because the factors of 15 (1,3,5,15), and the factors of 28 (1,2,4,7,14,28) are not in common (except for 1).

Twin Primes: A pair of prime numbers that differ by 2 (successive odd numbers that are both Prime numbers).

For Example: (3,5), (5,7), (11,13), ...

Numbers at a glance

Example	Number type
0.45	rational, real
3.1415926535....	irrational, real
3.14159	rational, real
0	whole, integer, rational, real
$\frac{5}{3}$	rational, real
$1\frac{2}{3}=\frac{5}{3}$	rational, real
$\sqrt{2}=1.41421356$	irrational, real
$-\sqrt{81}=-9$	integer, rational, real
-3	rational, real
$\sqrt{25}=5$	natural, whole, integer, rational, real
9/3 = 3	natural, whole, integer, rational, real
–0.75	rational, real
$\pi = 3.1428571\ldots$	irrational, real
3.144444.......	rational, real (since it is a repeating decimal)
$\sqrt{-9}$	Imaginary

PLACE VALUE AND FACE VALUE

In decimal number system, the value of a digit depends on its place or position in the number. Each place has a value of 10 times the place to its right.

Place value : Place value is a positional system of notation in which the position of a number with respect to a point determines its value. In the decimal system, the value of the digits is based on the number ten.

Each position in a decimal number has a value that is a power of 10. A decimal point separates the non-negative powers of 10, $(10)^0=1$, $(10)^1=10$, $(10)^2=100$, $(10)^3=1000$, etc.) on the left from the negative powers of 10, $(10)^{-1}=\frac{1}{10}$, $(10)^{-2}=\frac{1}{100}$, $(10)^{-3}=\frac{1}{1000}$, etc.) on the right.

Face value : The face value of a number is the value of the number without regard to where it is in another number. So 4 7 always has a face value of 7. However the place value includes the position of the number in another number. So in the number 4,732, the 7 has a place value of 700, but has a face value of just 7.

***Example*: Place and face values of the digits in the number 495, 784:**

Number	Digit	Place value	Face value
495,784	4	400000	4
	9	90000	9
	5	5000	5
	7	700	7
	8	80	8
	4	4	4

NAMES OF DIGITS ACCORDING TO THEIR PLACE VALUE.

Indian Method	International Method		
Unit	Unit	**1**	**1**
Ten	Ten	**10**	10^1
Hundred	Hundred	**100**	10^2
Thousand	Thousand	**1000**	10^3
Ten thousand	Ten thousand	**10000**	10^4
Lakh	Hundred thousand	**100000**	10^5
Ten lakh	One million	**1000000**	10^6
Crore	Ten million	**10000000**	10^7
Ten crore	Hundred million	**100000000**	10^8
Arab	Billion	**1000000000**	10^9

FRACTIONS

A fraction is known as a rational number and written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. The lower number 'q' is known as denominator and the upper number 'p' is known as numerator.

Type of Fractions :

Proper Fraction: The fraction in which numerator is less than the denominator is called a proper fraction.

For Example: $\frac{2}{3}, \frac{5}{6}, \frac{10}{11}$ etc.

Improper fraction : The fraction in which numerator is greater than the denominator is called improper fraction.

For Example : $\frac{3}{2}, \frac{6}{5}, \frac{8}{7}$, etc

Mixed fraction : Mixed fraction is a composite of a fraction and a whole number.

For example: $2\frac{1}{2}, 3\frac{3}{4}, 5\frac{6}{7}$ etc.

Complex fraction: A complex fraction is that fraction in which numerator or denominator or both are fractions.

For Example: $\frac{\frac{2}{3}}{4}, \frac{\frac{2}{5}}{\frac{6}{7}}, \frac{\frac{3}{7}}{\frac{5}{6}}$, etc.

Decimal fraction: The fraction whose denominator is 10 or its higher power, is called a decimal fraction.

For Example: $\frac{7}{10}, \frac{11}{100}, \frac{12}{1000}$

Continued fraction: Fractions which contain addition or subtraction of fractions or a series of fractions generally in denominator (sometimes in numerator also) are called continued

These are It is also defined as fractions whose numerator is an integer and whose denominator is an integer plus a fraction.

For Example: $2-\frac{\frac{2}{2}}{2+\frac{3}{4}}$

Comparison of Fractions :

- **If the denominators of all the given fractions are equal then the fraction with greater numerator will be the greater fraction.**

  For Example: $\frac{4}{7}, \frac{2}{7}, \frac{8}{7}, \frac{9}{7}$

  then, $\frac{9}{7} > \frac{8}{7} > \frac{4}{7} > \frac{2}{7}$

- **If the numerators of all the given fractions are equal then the fraction with smaller denominator will be greater fraction.**

  For Example: $\frac{7}{4}, \frac{7}{2}, \frac{7}{8}, \frac{7}{9}$ then, $\frac{7}{2} > \frac{7}{4} > \frac{7}{8} > \frac{7}{9}$

- **When numerator is greater than denominator and the differences of numerator and denominator are equal, then the fraction with smaller numerator will be the greater faction.**

  For Example: $\frac{5}{2}, \frac{7}{4}, \frac{11}{8}, \frac{8}{5}$

  then, $\frac{5}{2} > \frac{7}{4} > \frac{8}{5} > \frac{11}{8}$

Quicker Method (Cross Multiplication) :

This is a shortcut method to compare fractions. Using this method we can compare all types of fractions.

$$\frac{5}{9} \; ? \; \frac{4}{7}$$

$$5 \times 7 = 35 \qquad 9 \times 4 = 36$$

The fraction whose numerator is in the greater product is greater.

Since 36 is greater than 35, hence, $\frac{4}{7} > \frac{5}{9}$

LCM AND HCF

Factors and Multiples : If a number x divides another number y exactly, we say that x is a factor of y. Also y is called a multiple of x.

Highest Common Factor (HCF) :

The H.C.F. of two or more than two numbers is the greatest number that divides each one of them exactly. There are two methods for determining H.C.F.:

1. **Prime factorization method :** We can determine the H.C.F. of 144, 180 and 108 from following process.

$$144 = \underline{2 \times 2} \times 2 \times 2 \times \underline{3 \times 3}$$
$$108 = \underline{2 \times 2} \times \underline{3 \times 3} \times 3$$
$$180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$$

In prime factoriation of the above mentioned three numbers, the common factorization is $2 \times 2 \times 3 \times 3 = 36$.
Thus, required H.C.F. of 144, 180 and 108 is 36.

2. **Division Method:** We can determine the H.C.F. of above mentioned numbers from the following process :

$$
\begin{array}{r|l|l}
144 & 180 & 1 \\
 & 144 & \\
\hline
36 & 144 & 4 \\
 & 144 & \\
\hline
 & \times\times\times &
\end{array}
$$

Thus, the H.C.F of 144 and 180 is 36.
Now, we find the H.C.F of 36 and 108.

$$
\begin{array}{r|l|l}
36 & 108 & 3 \\
 & 108 & \\
\hline
 & \times\times\times &
\end{array}
$$

So, required H.C.F is 36.

Lowest Common Multiple (LCM) :

The L.C.M. of two or more than two numbers is the least number which is exactly divisible by each one of the given numbers.

❖ **Formula**: Product of two numbers
$= (\text{their H.C.F.}) \times (\text{their L.C.M.})$.

We can determine L.C.M. of two given numbers by the following two methods:

1. **Prime Factorization method:** Suppose we have to find the L.C.M. of 12, 16 and 30, then

$$12 = 2 \times 2 \times 3$$
$$16 = 2 \times 2 \times 2 \times 2$$
$$30 = 2 \times 3 \times 5$$

Thus, required L.C.M. of the given numbers
$= \underline{2 \times 2 \times 2 \times 2 \times 2} \times \underline{3} \times \underline{5} = 240$

2. **Division method:** We can determine the L.C.M. of above mentioned numbers from the following process :

$$
\begin{array}{r|l}
2 & 12,\ 16,\ 30 \\
\hline
2 & 6,\ 8,\ 15 \\
\hline
3 & 3,\ 4,\ 15 \\
\hline
 & 1,\ 4,\ 5
\end{array}
$$

Thus, required L.C.M. of the given number
$= 2 \times 2 \times 3 \times 1 \times 4 \times 5 = 240$

H.C.F. and L.C.M. of Fractions:

❖ H.C.F. of factions $= \dfrac{\text{H.C.F.of Numerators}}{\text{L.C.M.of Denominators}}$

For Example, we have to find the H.C.F. of $\frac{1}{2}$ and $\frac{3}{4}$.

Then, H.C.F. of $\frac{1}{2}$ and $\frac{3}{4} = \dfrac{\text{H.C.F.of 1 and 3}}{\text{L.C.M.of 2 and 4}} = \dfrac{1}{4}$

❖. L.C.M of fractions $= \dfrac{\text{L.C.M.of Numerators}}{\text{H.C.F.Denominators}}$

For Example, we have to find the L.C.M. of $\frac{1}{2}$ and $\frac{3}{4}$.

Then, L.C.M. of $\frac{1}{2}$ and $\frac{3}{4} = \dfrac{\text{L.C.M.of 1 and 3}}{\text{H.C.F.of 2 and 4}} = \dfrac{3}{2}$

Formulae to Remember

❖ The product of two numbers = (HCF of the numbers) × (LCM of the numbers)

❖ Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$

❖ Sum of first n even numbers $= \dfrac{\text{Last even number (last even number + 2)}}{4}$

❖ Sum of first n odd numbers $= \left(\dfrac{\text{last odd number} + 1}{2}\right)^2$

In the sequence, A, A + D, A + 2D, A + 3D .... Nth term = A + (N – 1) D

and sum of N terms $= \dfrac{N}{2}[2A + (N-1)D]$

Rules of Divisibility

These rules let you test if one number can be evenly divided by another, without having to do too much calculation!

(Divisibility Conditions)

A number is divisible by	If	Example
2	The last digit is even (0, 2, 4, 6, 8 …..)	12**8** is 129 is not
3	The sum of the digits is evenly/ completely divisible by 3	381 (3 + 8 + 1 = 12, and 12 ÷ 3 = 4) **Yes** 217 (2 + 1 + 7 = 10, and 10 ÷ 3 = $3\,^1/_3$) **No**
4	The last 2 digits are evenly/ completelydivisible by 4	13**12**, (12 ÷ 4 = 3) is 70**19** is not
5	The last digit is 0 or 5	17**5** is 809 is not
6	The number is evenly / completely divisible by both 2 and 3	114 (it is even and 1 + 1 + 4 = 6 and 6 ÷ 3 = 2) Yes 308 (it is even but 3 + 0 + 8 = 11 and 11 ÷ 3 = $3\frac{2}{3}$) **No**
7	If you double the last digit and subtract it from the rest of the number and the answer is : **0 or divisible by 7** (Note : for bigger numbers you can apply this rule to the answer again if you want)	672 (Double 2 is 4, 67 - 4 = 63, and 63 ÷ 7 = 9) **Yes** 905 (Double 5 is 10 , 90 - 10 = 80, and 80 ÷ 7 = 11 3/7) No
8	The last three digits are divisible by 8	109**816** (816 ÷ 8 = 102) **Yes** 216**302** (302 ÷ 8 = 37 3/4) **No**
9	The sum of the digits is divisible by 9 (Note : for bigger numbers you can apply this rule to the answer again if you want)	1629 (1 + 6 + 2 + 9 = 18, and again, 1 + 8 = 9) **Yes** 2013 (2 + 0 + 1 + 3 = 6) **No**
10	The number ends in 0	22**0** is 221 is not
11	If the difference of the sum of the digits at odd places and the sum of the digits at even places is **0 or divisible by 11**	**1364** ((3 + 4) – (1 + 6) = **0**) **Yes** **25176** ((5 + 7) – (2 + 1 + 6) = **3**) **No**
12	(i) The number is divisible by 3 and 4 both, or (ii) If you subtract the last digit from twice the rest of the number and the answer is : **0 or divisible by 12** (Note : for bigger numbers this can be applied repeatedly)	648 (6 + 4 + 8 = 18 and 18 ÷ 3 = 6, and 48 ÷ 4 = 12) **Yes** 916 (9 + 1 + 6 = 16, 16 ÷ 3 = $5\frac{1}{3}$) **No**

SOLVED EXAMPLES

EXAMPLE 1 : If an amount of ₹ 198011 is distributed equally amongst 47 persons, how much amount would each person get ?

(a) **₹4123** (b) **₹4231**
(c) **₹4213** (d) **₹4132**
(e) **None of these**

Sol. (c) Sum received by each person = ₹ $\left(\frac{198011}{47}\right)$ = ₹4213

EXAMPLE 2 : A company canteen requires 798 bananas per week. Totally how many bananas will it require for the months of January, February and March, 2008 if the number of employees did not change during this period ?

(a) **10480** (b) **10277**
(c) **10586** (d) **10374**
(e) **None of these**

Sol. (d) Number of days in the months of January, February and March in 2008

$= 31 + 29 + 31 = 91$ days $= 91 \div 7$ weeks $= 13$ weeks

$\because$ Consumption of bananas in 1 week = 798

$\therefore$ Consumption of bananas in 13 weeks

$= 13 \times 798 = 10374$

EXAMPLE 3 : The cost of 2 rings and 4 bangles is ₹ 46854. What is the cost of 5 rings and 10 bangles ?

(a) **₹ 115345** (b) **₹ 117135**
(c) **₹ 116675** (d) **Cannot be determined**
(e) **None of these**

Sol. (b) Let the CP of 1 ring and 1 bangle be ₹ x and ₹ y respectively.

$2x + 4y = 46854$

$\Rightarrow 2.5\,(2x + 4y) = 2.5 \times 46854$

$\Rightarrow 5x + 10y =$ ₹ 117135

EXAMPLE 4 : If the sum of four consecutive even numbers is 228, which is the smallest of the numbers ?

(a) **52** (b) **54**
(c) **56** (d) **48**
(e) **None of these**

Sol. (b) According to the question,

$x + x + 2 + x + 4 + x + 6 = 228$

$\Rightarrow 4x + 12 = 228$

$\Rightarrow 4x = 228 - 12 = 216$

$\therefore\ x = \frac{216}{4} = 54$

$\therefore$ The smallest even number = 54

EXAMPLE 5 : The difference between a two-digit number and the number obtained after interchanging the two digits of the two- digit number is 27. The sum of the two digits of the two-digit number is 15. What is the two-digit number ?

(a) **87** (b) **96**
(c) **69** (d) **Cannot be determined**
(e) **None of these**

Sol. (d) Let the two digit nubmer be 10x + y, where x is the first digit and y the second digit.

$\therefore\ (10x + y) - (10y + x) = 27$

$9x - 9y = 27$

$x - y = 3$ ....(a)

also $x + y = 15$ ....(b)

$\therefore\ x = 9$ and $y = 6$

$\therefore$ Required number is 96 or 69

EXAMPLE 6 : Five bells begin to toll together at intervals of 9 seconds, 6 seconds, 4 seconds, 10 seconds and 8 seconds respectively. How many times will they toll together in the span of one hour (excluding the toll at the start)?

(a) **5** (b) **8**
(c) **10** (d) **Cannot be determined**
(e) **None of these**

Sol. (c)

2	9,	6,	4,	10,	8
2	9,	3,	2,	5,	4
3	9,	3,	1,	5,	2
	3,	1,	1,	5,	2

$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$ sec.

$= \frac{1}{10}$ Hour.

The bells will toll together after an interval of $\frac{1}{10}$ hour.

$\therefore$ they will toll together 10 times in $\frac{1}{10}$ hour.

EXAMPLE 7 : Samantha, Jessica and Roseline begin to jog around a circular stadium. They complete their one lap around the stadium in 84 seconds, 56 seconds and 63 seconds respectively. After how many seconds will they be together at the starting point?

(a) **336** (b) **504**
(c) **252** (d) **Cannot be determined**
(e) **None of these**

Sol. (b) LCM of 84, 56, 63

$$\begin{array}{c|l} 2 & 84,\ 56,\ 63, \\ \hline 2 & 42,\ 28,\ 63, \\ \hline 7 & 21,\ 14,\ 63, \\ \hline 3 & 3,\ \ 2\ \ \ 9 \\ \hline & 1,\ \ 2,\ \ 3, \end{array}$$

$\therefore 2 \times 2 \times 7 \times 3 \times 2 \times 3 = 504$

Hence, all three persons will be together at the starting point after 504 seconds.

EXAMPLE 8 : If the fractions $\frac{2}{5}, \frac{3}{8}, \frac{4}{9}, \frac{5}{13}$ and $\frac{6}{11}$ are arranged in ascending order of their values, which one will be the fourth?

(a) $\frac{4}{9}$ **(b)** $\frac{5}{13}$

(c) $\frac{3}{8}$ **(d)** $\frac{2}{5}$

(e) None of these

Sol. (a) $\frac{2}{5} = 0.4,$ $\frac{3}{8} = 0.375,$

$\frac{4}{9} = 0.44,$ $\frac{5}{13} = 0.38,$

$\frac{6}{11} = 0.54$

$\therefore$ Ascending order is

$= \frac{3}{8}, \frac{5}{13}, \frac{2}{5}, \frac{4}{9}, \frac{6}{11}$

So the fourth one will be $\frac{4}{9}$.

EXAMPLE 9 : Bhuvan has some hens and some cows. If the total number of animal-heads are 71 and the total number of feet are 228, how many hens does Bhuvan have ?

(a) 43 **(b) 32**

(c) 24

(d) Cannot be determined

(e) None of these

Sol. (e) Let Bhuwan have x hens and y cows

According to the question,

$x + y = 71$ ...(i)

$2x + 4y = 228$ ...(ii)

Multiply equation (i) by 4 and subtract equation (ii) from it :

$4x + 4y - 2x - 4y = 284 - 228$

or, $2x = 56$

or, $x = \frac{56}{2} = 28$

$\therefore$ Number of hens = 28

EXAMPLE 10 : $\frac{1}{4}$ th of $\frac{2}{5}$ th of a number is 82. What is the number?

(a) 410 **(b) 820**

(c) 420 **(d) 220**

(e) None of these

Sol. (b) Let the number be $= x$

According to the question,

$$x \times \frac{2}{5} \times \frac{1}{4} = 82$$

$$\text{or, } x = \frac{82 \times 5 \times 4}{2} = 820$$

EXERCISE

1. What is 456 times 121?
   (a) 56453 (b) 54167
   (c) 55176 (d) 54155
   (e) None of these
2. The product of two consecutive even numbers is 12768. What is the greater number ?
   (a) 110 (b) 108
   (c) 114 (d) 112
   (e) None of these
3. An amount of ₹ 50176 is distributed equally amongst 32 persons. How much amount would each person get?
   (a) ₹ 1,555 (b) ₹ 1,478
   (c) ₹ 1,460 (d) ₹ 1,568
   (e) None of these
4. If an amount of ₹ 1,72,850 is equally distributed amongst 25 people, how much amount would each person get ?
   (a) ₹ 8912.50 (b) ₹ 8642.50
   (c) ₹ 7130 (d) ₹ 6914
   (e) None of these
5. The sum of four consecutive even numbers. A, B, C, and D is 180. What is the sum of the set of next four consecutive even numbers ?
   (a) 214 (b) 212
   (c) 196 (d) 204
   (e) None of these
6. What is 786 times 964 ?
   (a) 759276 (b) 749844
   (c) 75416 (d) 757704
   (e) None of these
7. The difference between a two-digit number and the number obtained by interchanging the two digits of the number is 18. The sum of the two digits of the number is 12. What is the product of the two digits of the two digits number ?
   (a) 35 (b) 27
   (c) 32 (d) Cannot be determined
   (e) None of these
8. What is 783 times 869?
   (a) 678689 (b) 678861
   (c) 680427 (d) 681993
   (e) None of these
9. There are 15 dozen candles in a box. If there are 39 such boxes. How many candles are there in all the boxes together?
   (a) 7020 (b) 6660
   (c) 6552 (d) 3510
   (e) None of these
10. Monica, Veronica and Rachael begin to jog around a circular stadium. They complete their one lap in 48 seconds, 64 seconds and 72 seconds respectively. After how many seconds will they be together at the starting point ?
    (a) 336 (b) 252
    (c) 576 (d) Cannot be determined
    (e) None of these
11. The product of two consecutive odd numbers is 19043. Which is the smaller number?
    (a) 137 (b) 131
    (c) 133 (d) 129
    (e) None of these
12. What is 131 times 333 ?
    (a) 46323 (b) 43623
    (c) 43290 (d) 44955
    (e) None of these
13. The product of two successive numbers is 8556. What is the smaller number?
    (a) 89 (b) 94
    (c) 90 (d) 92
    (e) None of these
14. A canteen requires 112 kgs of wheat for one week. How many kgs of wheat will it require for 69 days?
    (a) 1,204kgs (b) 1,401kgs
    (c) 1,104kgs (d) 1,014kgs
    (e) None of these
15. If an amount of Rs 41,910 is distributed equally amongst 22 persons, how much amount would each person get ?
    (a) ₹1905 (b) ₹2000
    (c) ₹745 (d) ₹765
    (e) None of these
16. The product of two consecutive even numbers is 4488. Which is the smaller number?
    (a) 62 (b) 71
    (c) 66 (d) 65
    (e) None of these
17. A canteen requires 21 dozen bananas for one week. How many dozen bananas will it require for 54 days?
    (a) 162 (b) 1944
    (c) 165 (d) 2052
    (e) None of these